Low-Temperature Galvanomagnetic Phenomena in an Intense Electric Field

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(Received 2 December 1963; revised manuscript received 16 January 1964)

The electron distribution function is calculated for a many-valley semiconductor in an intense electric field for the case of anisotropic scattering by acoustical phonons at low temperatures.

I. INTRODUCTION

IN this paper we shall consider the low-temperature galvanomagnetic properties of a semiconductor in an intense electric field. In order to calculate the low-temperature anisotropic transport properties of semiconductors such as n-type germanium and silicon, it is necessary to abandon both the simple picture of spherical constant energy surfaces and the assumption of phonon equipartition.

Zylberstejn and Conwell[1] have studied the deviations of the phonon distribution from equilibrium in a semiconductor in an intense electric field at low temperatures. They show that the deviation is small for a sample of small dimensions and small carrier concentration. We shall treat this latter case and assume that the phonon distribution remains in equilibrium.

The predominant electron scattering mechanism in a pure semiconductor at low temperatures is by acoustical phonons and the equilibrium number of such phonons of crystal momentum $\mathbf{q}$ is given by

$$N_q = 1/(e^{qs/kT}-1), \qquad (1)$$

where s is the velocity of sound, k is Boltzmann's constant, and T is the absolute temperature. The conservation of energy and crystal momentum require that the acoustical phonons with which an electron of energy ϵ can interact have a maximum energy of the order of $2(\epsilon ms^2)^{1/2}$ where m is the effective electron mass. At high temperatures the average electron energy $\bar{\epsilon}$, even in an intense electric field, is such that

$$2(\bar{\epsilon}ms^2)^{1/2} \ll kT \quad \text{and therefore} \quad qsN_q \approx kT. \qquad (2)$$

This is the case of equipartition of energy.

At low temperatures, on the other hand, an intense electric field may result in the average electron energy $\bar{\epsilon}$ being sufficiently large such that

$$2(\bar{\epsilon}ms^2)^{1/2} > kT \quad \text{and} \quad N_q \sim \exp-\left(\frac{2(\bar{\epsilon}ms^2)^{1/2}}{kT}\right). \qquad (3)$$

Since the probability of phonon emission and absorption are proportional to $1+N_q$ and N_q, respectively, we may in this case neglect N_q compared to one and consider only acoustical phonon emission; this is the case of zero-point scattering.

Stratton[2] has calculated the electron distribution function in the limit of zero-point scattering for the case of spherical constant energy surfaces and isotropic scattering, while Paranjape[3] has calculated the electron temperature for a Boltzmann distribution for the same case. Koenig, Brown, and Schillinger (K.B.S.)[4] have recently applied Shibuya's[5] theory of hot electrons in order to describe hot electron phenomena in n-type germanium at low temperatures. Shibuya's treatment is based on the assumption of phonon equipartition and is therefore not applicable under the condition for which K.B.S. have used it. We see in fact from Table II of their paper that the electron temperatures, or the corresponding mean electron energies, do not satisfy the equipartition condition (2).

Stratton's zero-point distribution function and Paranjape's calculation predict an electron mobility $\mu \sim E^{-0.8}$, where E is the electric field. This is in agreement with the low-temperature conductivity measurements of Bray and Brown.[6]

In this paper we shall extend the previous calculations[2,3] by considering a many-valley semiconductor with ellipsoidal constant energy surfaces in an intense electric field and a magnetic field. We treat the case of acoustical phonon scattering in the zero-point limit, and allow for anisotropic scattering in our calculation.

In Sec. II of this paper we set up the Boltzmann equation for a many-valley semiconductor, and in Sec. III we derive expressions for the energy and momentum relaxation in the case of anisotropic zero-point scattering. We solve the Boltzmann equation for the distribution function in Sec. IV.

II. BASIC EQUATIONS

In a previous publication,[7] the author has shown that the Boltzmann equation for the case of ellipsoidal constant energy surfaces can be expressed as two coupled equations for S and A, the isotropic and aniso-

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[5] M. Shibuya, Phys. Rev. **99**, 1189 (1955).
[6] R. Bray, D. Brown, *Proceedings of the International Conference on Semiconductor Physics, Prague, 1960* (Czechoslovakian Academy of Sciences, Prague, 1961).
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tropic parts of the distribution function, respectively,

$$e\mathbf{E}'\cdot\nabla_{p'}S+\mathrm{An}\{e\mathbf{E}'\cdot\nabla_{p'}A\}+\frac{e}{m_0}\mathbf{p}'\times\mathbf{B}'\cdot\nabla_{p'}A=\hat{C}'A\,,$$

$$\mathrm{Is}\{e\mathbf{E}'\cdot\nabla_{p'}A\}=\hat{C}'S\,,\tag{4}$$

with $\mathbf{p}'=\alpha\mathbf{p}$, $\mathbf{E}'=\alpha\mathbf{E}$, $\mathbf{B}'=R\mathbf{B}$, $\epsilon=p'^2/2m_0$,

$$\alpha=\begin{bmatrix}\dfrac{m_0}{m_x} & 0 & 0\\ 0 & \dfrac{m_0}{m_y} & 0\\ 0 & 0 & \dfrac{m_0}{m_z}\end{bmatrix}^{1/2}\qquad R=\begin{bmatrix}m_x & 0 & 0\\ 0 & m_y & 0\\ 0 & 0 & m_z\end{bmatrix}^{1/2}\frac{m_0}{(m_xm_ym_z)^{1/2}},\tag{5}$$

where $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic field vectors, ϵ and $\mathbf{p}$ are the electron energy and momentum and m_x, m_y, m_z are the electron masses corresponding to the three principal axes of the ellipsoid.

An$\{x\}$ and Is$\{x\}$ represent the anisotropic and isotropic parts of x respectively, and $\hat{C}'$ represents the collision operator in $\mathbf{p}'$ space.

Assuming that $\hat{C}'A$ can be described by an energy-dependent tensor whose principal axes coincide with those of the ellipsoid:

$$\tau=\begin{bmatrix}\tau_x(\epsilon) & 0 & 0\\ 0 & \tau_y(\epsilon) & 0\\ 0 & 0 & \tau_z(\epsilon)\end{bmatrix},\tag{6}$$

and neglecting the An$\{e\mathbf{E}'\cdot\nabla p'A\}$ term (see Appendix II), we obtain straightforwardly:

$$A=-\frac{dS}{d\epsilon}\mathbf{V}'\cdot\left[\frac{\tau(e\mathbf{E}'+\tau e\mathbf{E}'\times e\mathbf{B}'/m_0)+(e^2/m_0^2)\mathbf{B}'(\mathbf{B}'\cdot e\mathbf{E}')\bar{\tau}^3}{1+(e^2/m_0^2)\bar{\tau}^3[(B_x'^2/\tau_x)+(B_y'^2/\tau_y)+(B_z'^2/\tau_z)]}\right],\tag{7}$$

$$\hat{C}'S=-\frac{2}{3m_0\epsilon^{1/2}}\frac{d}{d\epsilon}\left[\frac{dS}{d\epsilon}\epsilon^{3/2}\left(\frac{e^2\mathbf{E}'\tau\mathbf{E}'+\bar{\tau}^3(e^2\mathbf{E}'\cdot\mathbf{B}'/m_0)^2}{1+(\bar{\tau}^3e^2/m_0^2)[(B_x'^2/\tau_x)+(B_y'^2/\tau_y)+(B_z'^2/\tau_z)]}\right)\right],\tag{8}$$

where $\mathbf{V}'=(\mathbf{p}'/m_0)$ and $\bar{\tau}^3=\tau_x\tau_y\tau_z$.

We shall consider the solution of this equation after having calculated the energy relaxation term $\hat{C}'S$, and the relaxation time tensor τ.

III. CALCULATION OF ENERGY RELAXATION TERM AND RELAXATION TIME TENSOR

In this section we shall derive expressions for τ and $\hat{C}'S$ for a many-valley semiconductor with anisotropic scattering by the zero-point acoustical phonons. Denoting by M_L^2, c_L and M_T^2, c_T, the matrix elements and elastic constants corresponding to the longitudinal and transverse modes respectively, we obtain

$$\hat{C}f(\mathbf{p})=\frac{2\pi}{\hbar(2\pi\hbar)^3}\left\{\int d\mathbf{q}[f(\mathbf{p}+\mathbf{q})(1+N_q)-f(\mathbf{p})N_q][X_L+X_T]+\int d\mathbf{q}[f(\mathbf{p}-\mathbf{q})N_q-f(\mathbf{p})(1+N_q)][Y_L+Y_T]\right\};\tag{9}$$

$$X_{L,T}=(qs_{L,T}/2c_{L,T})M_{L,T}^2\delta(\epsilon(\mathbf{p}+\mathbf{q})-\epsilon(\mathbf{p})-qs_{L,T}),$$

where

$$Y_{L,T}=(qs_{L,T}/2c_{L,T})M_{L,T}^2\delta(\epsilon(\mathbf{p}-\mathbf{q})-\epsilon(\mathbf{p})+qs_{L,T}),$$

$\rho s_L^2=c_L$, $\rho s_T^2=c_T$ and where ρ is the density. The matrix elements have been calculated by Herring and Vogt,[8] and we see from Table V of their paper that M_L^2 and M_T^2 are simple polynomials in $\cos^2\theta$, where θ is the angle between the principal axis of the ellipsoid and $\mathbf{q}$.

Transforming the elliposidal constant energy surfaces to spheres (5) in both the electron and phonon space,

[8] C. Herring and E. Vogt, Phys. Rev. **101**, 944 (1956).

we obtain

$$\hat{C}'f=\frac{2\pi m_T m_L^{1/2}}{\hbar(2\pi\hbar)^3 m_0^{3/2}}\left\{\int d\mathbf{q}'[f(\mathbf{p}'+\mathbf{q}')(1+N_{q*})-f(\mathbf{p}')N_{q*}][X_L'+X_T'] + \int d\mathbf{q}'[f(\mathbf{p}'-\mathbf{q}')N_{q*}-f(\mathbf{p}')(1+N_{q*})][Y_L'+Y_T']\right\}, \quad (10)$$

where

$$X_{L,T}'=(q^*s_{L,T}/2c_{L,T})M_{L,T}^2(\cos^2\theta^*)\times\delta(\epsilon(\mathbf{p}'+\mathbf{q}')-\epsilon(\mathbf{p}')-q^*s_{L,T}),$$

$$Y_{L,T}'=(q^*s_{L,T}/2c_{L,T})M_{L,T}^2(\cos^2\theta^*)\,\delta(\epsilon(\mathbf{p}'-\mathbf{q}')-\epsilon(\mathbf{p}')+q^*s_{L,T}).$$

The principal axis of the ellipsoid has been taken as the polar axis in $\mathbf{q}'$ space; and m_T and m_L are the transverse and longitudinal masses, respectively;

$$q^*=(m_T/m_0)^{1/2}q'[1+(K-1)\cos^2\theta']^{1/2},$$

$$\cos\theta^*=\frac{K^{1/2}\cos\theta'}{[1+(K-1)\cos^2\theta']^{1/2}},\quad K=\frac{m_L}{m_T}, \quad (11)$$

where θ' is the cone angle in $\mathbf{q}'$ space.

We first calculate $\hat{C}'S$, where S is a function of energy only. If the scattering were isotropic the matrix elements would be angularly independent and $\hat{C}'S$ would depend only on energy. In our case, M_L^2 and M_T^2 are angularly dependent and $\hat{C}'S$ will therefore in general be a function of the direction of $\mathbf{p}'$. We shall calculate the average of $\hat{C}'S$ over all directions of $\mathbf{p}'$, i.e.,

$$\langle\hat{C}'S\rangle_{\text{av}}=\int\int\hat{C}'Sd\Omega_{p'}\Big/\int\int d\Omega_{p'}, \quad (12)$$

where $d\Omega_{p'}$ is the differential solid angle in $\mathbf{p}'$ space.

Assuming that:

(1) The distribution function varies slowly over an energy interval equal to the phonon energy:

$$S(\epsilon+qs)=S(\epsilon)+qs\frac{dS}{d\epsilon}+\frac{(qs)^2}{2}\frac{d^2S}{d\epsilon^2}. \quad (13a)$$

(2) The average electron energy $\bar{\epsilon}$ is sufficiently large such that N_q is negligible compared to one:

$$2(\bar{\epsilon}ms^2)^{1/2}>kT \quad (13b)$$

and carrying out the required integrations we obtain

$$\hat{C}'S=\frac{64\pi^3 m_T^2 m_L^{1/2}}{\sqrt{2}h^4\rho\epsilon^{1/2}}\Sigma^{2*}\frac{d}{d\epsilon}\left[\epsilon^2\left(S+\frac{4\sqrt{2}}{5}m_T^{1/2}\epsilon^{1/2}V^*\frac{dS}{d\epsilon}\right)\right], \quad (14)$$

where

$$\Sigma^{2*}=\int_0^1 R^2\left[M_L^2\left(\frac{Kx^2}{R^2}\right)+M_T^2\left(\frac{Kx^2}{R_2}\right)\right]dx,$$

$$V^*=\int_0^1 R^3\left[s_L M_L^2\left(\frac{Kx^2}{R^2}\right)+s_T M_T^2\left(\frac{Kx^2}{R^2}\right)\right]dx\Big/\Sigma^{2*},$$

and

$$R=[1+(K-1)X^2]^{1/2}.$$

A relaxation time does not generally exist when the matrix elements are angularly dependent; we follow the procedure of Herring and Vogt and define two principal relaxation times as follows:

$$1/\tau_{\parallel}=-\int\int\phi_{\parallel}'\hat{C}'\phi_{\parallel}'d\Omega_{p'}\Big/\int\int\phi_{\parallel}'^2 d\Omega_{p'},$$

$$1/\tau_{\perp}=-\int\int\phi_{\perp}'\hat{C}'\phi_{\perp}'d\Omega_{p'}\Big/\int\int\phi_{\perp}'^2 d\Omega_{p'}, \quad (15)$$

with

$$\phi_{\parallel}'=\mathbf{p}'\cdot\mathbf{a}/p',\quad \phi_{\perp}'=\mathbf{p}'\cdot\mathbf{b}/p',$$

where $\mathbf{a}$ and $\mathbf{b}$ are unit vectors parallel to the major and minor axes of the ellipsoid, respectively. We obtain the following expressions for the two relaxation times:

$$\frac{1}{\tau_{\parallel,\perp}}=\frac{48(2\pi)^3}{5h^4}m_T^{3/2}m_L^{1/2}\epsilon N_{\parallel,\perp}, \quad (16)$$

where

$$N_{\parallel}=T_1,\quad N_{\perp}=\frac{T_0-T_1}{2},$$

and

$$T_n=\int_0^1 dxR(x)x^{2n}\left[\frac{s_L}{c_L}M_L^2\left(\frac{Kx^2}{R^2}\right)+\frac{s_T}{c_T}M_T^2\left(\frac{Kx^2}{R^2}\right)\right].$$

The matrix elements and average elastic constants are given in Table V of Herring and Vogt's paper:

$$M_L^2\left(\frac{Kx^2}{R^2}\right)=\Sigma_d^2\left(1+\frac{Kx^2W}{R^2}\right)^2\left[1+\gamma_L\left(0.15-\frac{1.5Kx^2}{R^2}+\frac{1.75K^2x^4}{R^4}\right)\right],$$

$$M_T{}^2\left(\frac{Kx^2}{R^2}\right)=\frac{Kx^2\Sigma_d{}^2W^2}{R^2}\left(1-\frac{Kx^2}{R^2}\right)\left(1+\gamma_T\frac{Kx^2}{R^2}\right);\quad W=\frac{\Sigma_\mu}{\Sigma_d},$$

$$c_L=c_{12}+2c_{44}+0.6c^*;\qquad \frac{1}{c_T}=\begin{cases}\dfrac{0.375}{c_{44}}+\dfrac{0.625}{c_{44}+\frac{1}{3}c^*}, & (111)\text{ valley}\\[2ex] \dfrac{3}{c_{44}+\frac{1}{3}c^*}-\dfrac{2}{c_{44}+\frac{1}{2}c^*}, & (100)\text{ valley},\end{cases}$$

$$\gamma_L=\begin{cases}\dfrac{2c^*}{3c_L}, & (111)\text{ valley}\\[2ex] -\dfrac{c^*}{c_L}, & (100)\text{ valley}\end{cases}\qquad \gamma_T=\begin{cases}\dfrac{9}{8}c_T\left(\dfrac{1}{c_{44}}-\dfrac{1}{c_{44}+\frac{1}{3}c^*}\right), & (111)\text{ valley}\\[2ex] 6c_T\left(\dfrac{1}{c_{44}+\frac{1}{2}c^*}-\dfrac{1}{c_{44}+\frac{1}{3}c^*}\right), & (100)\text{ valley},\end{cases}$$

where Σ_d and Σ_μ are the deformation potential constants for dilation and uniaxial shear. Inserting these matrix elements in Eqs. (16) and (14) we obtain the relaxation time tensor and $\hat{C}'S$ (see Appendix I).

IV. THE DISTRIBUTION FUNCTION

We shall now solve Eq. (8) for S, the isotopic part of the distribution function. Substituting Eq. (14) in Eq. (8), we obtain

$$S=\exp-\left[\int_0^\epsilon\frac{H\epsilon^{1/2}d\epsilon}{\dfrac{2}{3m_0}\left(\dfrac{e^2\mathbf{E}'\tau\mathbf{E}'+[(e^2/m_0)\mathbf{E}'\cdot\mathbf{B}']^2\bar{\tau}^3}{1+(e^2\bar{\tau}^3/m_0{}^2)[(B_x'^2/\tau_x)+(B_y'^2/\tau_y)+(B_z'^2/\tau_z)]}\right)+\bar{H}}\right],\tag{17}$$

where

$$H=\frac{64\pi^3m_T{}^2m_L{}^{1/2}\Sigma^{2*}}{\sqrt{2}h^4\rho};\quad \bar{H}=\frac{4\sqrt{2}}{5}m_T{}^{1/2}V^*\epsilon H.$$

We shall consider the case of a small magnetic field ($\omega^2\tau^2\ll1$) and we neglect the field-independent term in the denominator of the integrand of Eq. (17) (see Appendix II). We then obtain

$$S=\exp-(\epsilon/kT_e)^{5/2},\tag{18}$$

$$A=-\frac{dS}{d\epsilon}\mathbf{V}'\cdot\left[\tau\left(e\mathbf{E}'+e\tau\mathbf{E}'\times\frac{e\mathbf{B}'}{m_0}\right)\right],\tag{19}$$

with

$$T_e=X(\gamma)G\frac{[E\mu_aT^{3/2}/s_L]^{4/5}}{[m_Ts_L{}^2/k]^{1/5}}\left[\frac{3\bar{\Sigma}^4}{16\Sigma^{2*}N_\perp s_L\rho}\right]^{2/5},\tag{20}$$

where

$$X(\gamma)=\left[1+\gamma^2\left(\frac{1}{K'}-1\right)\right]^{2/5},\quad K'=K\frac{\tau_\perp}{\tau_{\parallel}},$$

$$G=\left[\frac{\sqrt{2}\pi(27)(25)}{16^2(2+1/K)^2}\right]^{2/5},$$

and γ is the direction cosine of $\mathbf{E}$ with respect to the longitudinal axis of the ellipsoid. μ_a is the low-field mobility for acoustical phonon scattering and is related to the average deformation potential which appears in Herring and Vogt's paper [Eqs. (49) and (50)]

$$\bar{\Sigma}^4=[\xi\Sigma_d{}^2+\eta\Sigma_\mu\Sigma_d+\zeta\Sigma_\mu{}^2]^2.$$

When we consider the simple model of spherical constant energy surfaces, $\Sigma_\mu=0$ and neglect elastic anisotropy ($c^*=0$), the last factor in our expression for T_e becomes equal to 1, as does $X(\gamma)$, and Eq. (18) becomes identical with the distribution function calculated by Stratton.[2] Since the low-field mobility for acoustical phonon scattering varies as $T^{-3/2}$, our expression T_e is independent of temperature.

Using Eqs. (19) and (20) we obtain the following expressions for the mobility μ and the low magnetic-field Hall coefficient R.

$$\mu=PE^{-0.8};\quad R=B,$$

where P and B are complicated functions of the orientation of the electric and magnetic fields with respect to the crystallographic axes but are independent of the *magnitudes* of the applied fields. The field dependence of the mobility is in agreement with the experimental results of Bray and Brown.[6] A discussion of the orientation dependence will be considered in a future publication.

ACKNOWLEDGMENTS

The author expresses his sincere thanks to Professor P. Aigrain and Dr. E. Conwell for their stimulating discussions. Thanks are also due to Dr. A. Zylberstejn and D. Olechna for helpful suggestions and to the Xerox Corporation for partial support of this work.

APPENDIX I

We shall express all the required integrals of Sec. III in terms of the elementary integrals

$$\langle m,n\rangle \equiv \int_0^1 \frac{x^m dx}{[1+(K-1)x^2]^{n/2}}, \tag{A1}$$

$$\begin{aligned}\Sigma^{2*}=\Sigma_d^2\{\tfrac{2}{3}+\tfrac{1}{3}K(1+W)^2+\gamma_L[0.1-0.45K+1.75K^2\langle 4,2\rangle+2KW(0.05-1.5K\langle 4,2\rangle+1.75K^2\langle 6,4\rangle)\\ +(KW)^2(0.15\langle 4,2\rangle-1.5K\langle 6,4\rangle+1.75K^2\langle 8,6\rangle)]+\gamma_T(KW)^2(\langle 4,2\rangle-K\langle 6,4\rangle)\}\,,\end{aligned} \tag{A2}$$

$$\begin{aligned}V^*=\frac{\Sigma_d^2}{\Sigma^{2*}}\{s_L(\langle 0,-3\rangle+2KW\langle 2,-1\rangle+K^2W^2\langle 4,1\rangle)+s_TKW^2(\langle 2,-1\rangle-K\langle 4,1\rangle)\\ +\gamma_Ls_L[0.15\langle 0,-3\rangle-1.5K\langle 2,-1\rangle+1.75K^2\langle 4,1\rangle+2KW(0.15\langle 2,-1\rangle-1.5K\langle 4,1\rangle+1.75K^2\langle 6,3\rangle)\\ +(KW)^2(0.15\langle 4,1\rangle-1.5K\langle 6,3\rangle+1.75K^2\langle 8,5\rangle)]+\gamma_Ts_T(KW)^2(\langle 4,1\rangle-K\langle 6,3\rangle)\}\,,\end{aligned} \tag{A3}$$

$$\begin{aligned}T_{\substack{0\\1}}=\Sigma_d^2\Big\{\frac{s_L}{c_L}(\langle \substack{0\\2},-1\rangle+2KW\langle \substack{2\\4},1\rangle+(KW)^2\langle \substack{4\\6},3\rangle)+\frac{s_T}{c_T}KW^2(\langle \substack{2\\4},1\rangle-K\langle \substack{4\\6},3\rangle)\\ +\gamma_L\frac{s_L}{c_L}[0.15\langle \substack{0\\2},-1\rangle-1.5K\langle \substack{2\\4},1\rangle+1.75K^2\langle \substack{4\\6},3\rangle+2KW(0.15\langle \substack{2\\4},1\rangle-1.5K\langle \substack{4\\6},3\rangle+1.75K^2\langle \substack{6\\8},5\rangle)\\ +(KW)^2(0.15\langle \substack{4\\6},3\rangle-1.5K\langle \substack{6\\8},5\rangle+1.75K^2\langle \substack{8\\10},7\rangle)]+\gamma_T\frac{s_T}{c_T}(KW)^2(\langle \substack{4\\6},3\rangle-K\langle \substack{6\\8},5\rangle)\Big\}\,.\end{aligned} \tag{A4}$$

The integrals $\langle m,n\rangle$ are all straightforward. For $K=1$, $\langle m,n\rangle=1/(m+1)$, and for $K\neq 1$ one can easily show that

$$\langle m+2,\, n+2\rangle=\frac{1}{n(K-1)}\left[(m+1)\langle m,n\rangle-\frac{1}{K^{n/2}}\right] \tag{A5}$$

for $n\neq 0$. We have tabulated some of the integrals in Table I. The remaining ones are easily calculated by means of Eq. (A5).

TABLE I. Integrals.

	$K>1$	$K=1$	$K<1$
$\langle 0,-1\rangle$	$\frac{K^{1/2}}{2}+\frac{\sinh^{-1}(K-1)^{1/2}}{2(K-1)^{1/2}}$	1	$\frac{K^{1/2}}{2}+\frac{\sin^{-1}(1-K)^{1/2}}{2(1-K)^{1/2}}$
$\langle 2,-1\rangle$	$\frac{1}{8(K-1)}\left[K^{1/2}(2K-1)-\frac{\sinh^{-1}(K-1)^{1/2}}{(K-1)^{1/2}}\right]$	$\frac{1}{3}$	$\frac{1}{8(1-K)}\left[-K^{1/2}(2K-1)+\frac{\sin^{-1}(1-K)^{1/2}}{(1-K)^{1/2}}\right]$
$\langle 4,2\rangle$	$\frac{1}{(K-1)^2}\left[\frac{K-4}{3}+\frac{\tan^{-1}(K-1)^{1/2}}{(K-1)^{1/2}}\right]$	$\frac{1}{5}$	$\frac{1}{(1-K)^2}\left[\frac{K-4}{3}+\frac{\tanh^{-1}(1-K)^{1/2}}{(1-K)^{1/2}}\right]$

APPENDIX II

We see from Eq. (18) that $\bar{\epsilon}=kT_e/\Gamma(0.6)\approx\frac{3}{2}kT_e$ and therefore using Eq. (3) the zero-point scattering condition is satisfied for electric fields large enough such that

$$\sqrt{6}\left(\frac{m^*s^{*2}}{k}\right)^{1/2}(X(\gamma)G)^{1/2}\frac{[E\mu_aT^{3/2}/s_L]^{2/5}}{[m_Ts_L^2/k]^{1/10}}\left[\frac{3\Sigma^4}{16\Sigma^{2*}N_\perp s_L\rho}\right]^{1/5}>T\,, \tag{A6}$$

where m^* is a complicated average of the longitudinal and transverse masses and is given approximately by

$$m^*\approx\tfrac{2}{3}m_T(K^{3/2}-1)/(K-1)$$

and s^* is the average sound velocity.

For spherical constant energy surfaces and $\Sigma_\mu=c^*=0$, Eq. (20) becomes approximately

$$2.4(ms^2/k)^{2/5}[E\mu_aT^{3/2}/s]^{2/5}>T\,.$$

The neglect of the field-independent term in the denominator of Eq. (17) is equivalent to neglecting ms^2/kT_e compared to one. It is easily seen that the $\mathrm{An}\{e\mathbf{E}'\cdot\nabla_{p'}A\}$ term is a correction to the distribution function due to neglecting higher order terms than the first-order spherical harmonics, i.e., terms of the form $Y_{2n}(\theta,\phi)$, etc.